What Conflicts Help Students Learn About Decimals?

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Sixteen Form 1 and 2 students worked in pairs to solve problems involving decimal fractions. Their discussions were analysed to see what kind of conflicts led to learning about the meaning of decimals. Useful conflicts arose from those written into the problems, differences between calculator results and expectations, discussions with a peer, and differences between out-of-school experience and calculations. The most profitable conflicts could be classified as those that brought about what Piaget called beta reactions.

Introduction

Provoking cognitive conflict to help students understand areas of mathematics is often recommended. It is a technique that teachers used in a recent research project and reported to be very effective, especially when used as a task in which students were given a sheet with some incorrect answers and asked to "correct homework" (Britt, Irwin, Ellis and Ritchie, 1993). Students discussed reasons why an answer was right or wrong, usually correcting mistakes that they were likely to have made themselves. Another study that found conflict more effective than direct instruction was that of Swan (1983, 1993) who taught one of two parallel classes by having students talk about the differences between their original responses on a decimal problem to the answers that they worked out using number lines.

The hypothesis that learning happens as the result of a learner being faced with conflict was emphasised by Piaget (e.g. 1975/1985). He cites Inhelder, Sinclair and Bovet (1974) as having shown that the most influential factor in acquiring new knowledge structures was the perturbation, or discomfort, resulting from unresolved conflict. This discomfort motivates learners to seek a resolution that incorporates both their existing knowledge and the new information.

Not all conflict is useful in teaching, however. If a student meets a conflict that shows that they are wrong and they cannot find out why, they may be led to believe that mathematics does not make sense, is too hard for them, or that they don't like it. McLeod (1995) applies Mandler's theory of interruption of a schema to suggest that what happens when students meet a mathematical conflict that they cannot resolve. What they remember in this case is their negative emotional reaction, which in time may lead to a negative attitude toward mathematics.

Piaget (1975/1985) provided a framework for classifying reactions to perturbations, or the disequilibrium caused by a new fact that conflicts with an existing scheme. He does not refer to the educational value of different responses to perturbation. However, his categories provide a way of examining the educational usefulness of students' reactions to conflicts.

In brief, Piaget described three categories of reaction. The first of these, an alpha reaction, does not lead to change in the learner's scheme because either (i) it involves adding a fact to an existing category without any need for modification or (ii) the new fact is denied because the learner cannot see a way of incorporating it. The second category of reaction, a beta reaction, does involve a change in the learner's concepts. A learner's initial scheme is altered in some way and the new fact is seen in a different way so that it is transformed through reciprocal assimilation and accommodation. In the third reaction, a gamma reaction, new facts do not cause perturbation because the learner already has a scheme that is flexible enough to incorporate it. Piaget acknowledges the contradiction inherent in calling this a reaction to a perturbation, but the category is necessary to cover the range of reactions of a student to conflict.

From an educational point of view, the gamma reaction could be seen as the goal of learning in a particular mathematical domain. It involves a having a thorough grasp of a domain. Within the classroom, however, it is the beta reactions that seem appropriate for teachers to aim for in the short term. These reactions to conflict provide the conceptual building blocks that go to make up an overall flexible scheme. Alpha reactions, on the other hand, are less useful in learning. Adding of new facts to an existing scheme may help solidify concepts but after a certain point becomes repetition of knowledge that is already secure. Denial of a new fact is of use in maintaining a learner's self confidence but does not extend knowledge. It could be argued that when students respond with the first of these alpha reactions the work being presented to the learner is too easy, and when the learner responds with the second of these alpha reactions the work is too hard.

If conflict is as important as Piaget (1975/1985) indicates, and as useful as an educational technique as found by Swan (1983, 1993), then it could be expected that facing and discussing conflicting interpretations of decimal fractions would help students improve their understanding. Working in pairs should increase the potential for conflict, as two learners might have different views. In the study reported here many more conflicts were observed than had been expected. These conflicts were discussed by the students at length, and usually resolved in an appropriate manner. Some attempts involved clinging to an initial belief but many involved the construction of new understanding. Perret-Clermont, Perret and Bell (1990) as well as Piaget point to hearing the different ideas of others as an important factor in deciding whether or not one's own ideas needed to be changed.

The Study

This study of students' reaction to conflict was only one aspect of a larger study of ways of enhancing students' understanding of decimal fractions. In this phase of the study 11- and 12-year-old students worked on problems involving decimal fractions in pairs that included a more and less competent student, as ranked by the teacher. All students took a pretest on their knowledge of decimal fractions. Then they worked in these pairs on three sets of problems on three separate days and their discussions were tape-recorded for transcription. These problems covered the magnitude of numbers, addition and subtraction, and multiplication and division. All problems involved decimal fractions in a format which forced students to think about the meaning of place and the decimal point. There were two sets of problems: those which were based on context and those that used a purely numerical format. Both contextualized and numerical problems were set up to maximise the likelihood of conflicting ideas occurring. Some problems presented two conflicting answers and asked to students to decide which was right, and some used conflict between answers that it was expected that the students would give and calculator answers. In each case students were required to say why they thought an answer was right.

One contextualized problem that presented two conflicting answers was:

> The paper says that one New Zealand dollar = 0.9309 in Australian dollars.
> Susan said that was 93.09 cents Why did she say that? Is she right?
> Andrew said it would be 9309 dollars. Why did he say that? Is he right?
> What do you think?

An example in which it was expected that students answers would conflict with calculator answers was:

> $1 New Zealand exchanges for 1.5989 Samoan tala.
> How much would you get for $10 New Zealand?
> Try it on your calculator. Why did it give that answer?

Similar purely numerical problems used numbers that were somewhat simpler, but reflected confusions shown in other research. Examples were:

> Teri said that $93\frac{1}{4}$ was written as 93.04 in decimals.

Why did she say that? Is she right?

Peta said that $93\frac{1}{4}$ was written as 93.25 in decimals

Why did he say that? Was he right? What do you think it should be?

How much is $1\frac{1}{2}$ x 10?

Try it on your calculator. Why did it give that answer?

All students were given a posttest four days after their last problem-solving session and again one month later. Students given contextualized problems showed significantly more gain, with an interaction between the progress of the groups given the different types of problems and pretest-posttest scores ($F(1,12)=5.70$, $p=.03$). One of the reason for their greater gain appeared to be related to the conflicts that they discussed.

Evidence of learning from conflict was shown by students given both types of problems and by both more and less knowledgeable students. Conflict arose from what was written in the problems, from conflict between student expectations and calculator results, from discrepant results between a calculation and expectations based on outside experience, from conflict between different results from manual calculation, from differences between the peers, and from discrepancies between a new conclusion and previous views. Some conflicts involved both students in a pair offering suggestions and building on each other's understanding to resolve the conflict. Other conflicts involved an individual student attempting to resolve a contradiction through repeated attempts to calculate or understand an issue, bringing different aspects of knowledge to bear. Students also responded to these conflicts in a variety of ways, only some of which were educationally useful. These conflicts can be described using all three of Piaget's categories, with several subdivisions some of which relate to the writing of others.

Reactions to Conflict

Responses That Did Not Appear to Lead to Learning (Alpha Reactions)

Recognising an inconsistency as a problem to others but not to them: This appeared to be an example of Piaget's alpha (i) reaction, in which students were able to add on a fact without reorganising their own understanding. In solving these problems students acted like those who took part in the "correcting homework" exercises mentioned above. For example, in one problem a hypothetical students said that the next number after 3.56 was 4.57 (as some students in a previous part of this study had said) and the other hypothetical student said that the next number after 3.56 was 3.57. The students given these problems weren't drawn into the intended conflicts but they could see what the students named in the problem might have thought which led them to give a wrong answer.

Ignoring: Ignoring a conflicting element is an example of Piaget's alpha (ii) reaction. Vinner (1990) also wrote about inconsistencies that students ignored because they did not understand the contradiction. This reaction was displayed by K who responded to the question .9 + .1 by reading 'nine plus one', predicting the answer 'ten' and then reading the result of 1.0 on the calculator as 10. By ignoring the decimal point he did not have to deal with any conflict in this problem. The more common response was to read this question as "point nine plus point one" and predict "point ten" as the answer. Students who did this were surprised when the calculator gave "one point oh" as the answer and worked to resolve that conflict, in beta reactions.

Recognising and denying the importance of a conflict by removing the disturbing element: This was another example of Piaget's alpha (ii) reaction, but differed from the one given above in that students acknowledged the conflict but then chose to resolve it by denying some aspect of the new fact. They reacted by choosing to disregard it or in some way removed what was disturbing.

An example of removing what was disturbing was shown by G and A. This was an unplanned conflict between manual calculation and calculation with a calculator. In multiplying .355 x 6 G repeatedly got the answer 2.103 because he was convinced that in the first multiplication (from the right) of 6 x 5 you put down the 3 and carried the 0, writing:

$$\begin{array}{r} .{}^{3}3\ {}^{0}5\ 5 \\ \underline{\mathrm{x}\qquad 6} \\ 2.1\ 0\ 3 \end{array}$$

The calculator gave the answer 2.13. His partner finally resolved this conflict by crossing out the medial 0 in this calculation, making the answers the same. G did not appear to agree but gave in with a written answer that blamed the calculator, saying "Because it (the calculator) took out the zeros". He thereby maintained his confidence in his own calculation by blaming an electronic tool for making a mistake.

Responses That Could Lead To Learning (Beta Reactions)

These were the reactions in which learners integrated the perturbing element into their scheme, either by extending their existing scheme or changing their scheme to include the perturbing element, or both. In these students' discussions, the disturbance caused by the conflicting element was resolved by either treating the new fact as applying to a particular context (compartmentalising), or by integrating it into an existing scheme with the help of knowledge from a different domain, thus extending their initial scheme.

Compartmentalising new knowledge: Duffin and Simpson (1993) write about the reaction in which a new fact is treated as applying to a particular context as treating a new idea as 'foreign', or unrelated to any currently held mathematical ideas. It is common for mathematicians writing about conflict to give experiences from their own lives in which they learned a piece of mathematical knowledge in a compartmentalised manner and only years later saw its relevance to other areas of mathematics. (e.g. Duffin & Simpson, 1993; Mavshovitz & Hadar, 1993; Vinner, 1990). Contextualized or compartmentalised understanding is an important first step to more advanced understanding

There were several instances of compartmentalising knowledge that appeared to be a beta reaction. One instance was shown by a student who thought of halves and quarters in terms of time with 60 minutes in an hour, so that a quarter was written .15 and a half was written as .30. His partner was quick to grasp the source of this compartmentalised knowledge and say that it was true for time, but that decimal fractions were based on 100, so a quarter was .25 and a half was .50. The student who had held an understanding compartmentalised in telling time was happy to see the truth of his situated knowledge and at the same time accept the nature of decimal fractions a different situation in which a quarter and a half meant something different. In this case it was the quarter and half that were the wider concept, and both time and decimal fractions were specific situations to which the concept could be applied.

The result shown by a calculator was the source of many of the conflicts experienced by students. In each pair there was at least one student who knew that calculators dropped off the final zero(s) in a decimal fractions. However, this knowledge appeared to be compartmentalised, limited to the behaviour of calculators, rather than being seen as a general principle that could have been applied manual calculation as well. For example G explained to B why the calculator gives 0.3 as the answer to 1.45 - 1.15 by explaining, correctly:

"Cus on the calculator, when it does that it just doesn't put, add the zero."

The other main type of compartmentalisation was attention to the digits or decimal point without attending to meaning. M and N showed this attention to symbols rather than to meaning when trying to explain why the hypothetical student in the problem read \$0.9309 as 93.09 cents. The dialogue suggested that they were struggling to understand this novel representation of money. As it was the representation that they were trying to understand it is not surprising that they attended to this. Later in their discussion, when faced with another conflict, they make a decision based on meaning.

K That's how the paper records exchange rates.
N Zero point 9 3 0 9.
M Oh, cause she rounded it up.(Pause)
N Did she? It's still the same, she just took off the zero, put a dot....
M Oh no she just put the decimal point two backwards.
N Took out the zero.

Correction: In discussing different types of conflict, Vinner (1990) makes a distinction between superficial and deep conflicts. Superficial conflicts are those that can be corrected by redoing a calculation. These conflicts may not seem superficial for the students who rework a problem several times before they find and correct their errors. The conflict is resolved by changing the incoming fact, or calculation, to match expectations. Vinner classifies corrections as superficial because once the correct calculation is found the student immediately knows that it is right. Deep contradictions on the other hand require a greater reorganisation of existing understanding before they can be resolved.

There were many examples of corrected calculations in finding solutions to these problems. Students' first reaction to contradictory answers was often to rework a problem both manually and on their calculator It was these calculation conflicts that the lower-ranked students were helpful in identifying and resolving. On one occasion a lower-ranked peer told her partner that she should be adding not subtracting, thus breaking an impasse. In another example M had mentally calculated that 3000 - 500 = 800, presumable because she was adding the significant digits rather than subtracting. Her partner, N, quickly corrected her saying:

N No not eight hundred.
M Three thousand... five hundred.
N One..no..two thousand five \
M Two thousand three, no two thousand five hundred, yeah.
N Yeah, two thousand five hundred.

Addition of a new fact: There were also cases in which one student's confusion was overcome by the partner offering the needed piece of information as a correction. One example occurred when one student wrote $90\frac{9}{10}$ as 90.910. His partner then informed him that $\frac{9}{10}$ meant "nine tenTHS". This appeared to be new information for the first student who then added it to his store of facts.

Conflicts That Were Solved by Changing the New Fact: Problems that were solved through seeing the relationship between measurement units fall in this category of changing an incoming fact. The problems involving both centimetres and millimetres, centimetres and metres, kilograms and grams, and kilometres and metres provoked discussion about the relationship of these units. The students asked each other how many of the smaller unit there were in the larger, and then changed the larger unit to a multiple of the smaller unit. This changing of the problem enabled students to fit it into their understanding without the need to deal with different units.

Conflicts between two contradictory facts both of which appear to fit the scheme: Several problems in this study were set up to provoke this type of conflict in the hope that students would see the error of the answer which involved a misconception and strengthen their understanding of the correct scheme. However, for some students this conflict was severe. Piaget would classify it as a conflict between two sub-procedures. Their reaction, which was to try to fit one or both facts into their larger scheme, would be a beta reaction. One example was a discussion, that ran to five single-spaced pages of transcript, was a debate about whether $90\frac{1}{4}$ was written as 90.04 or 90.25. The answer 90.04, fits a fraction misconception in which the number of parts something is divided into is given as the decimal fraction. The second answer, 90.25, fits a belief that the decimal fraction for a 1/4 is .25 because there are four 25s in 100. The participants provided a variety of reasons for each of the answers being correct, but H insisted on continuing to work on the problem until she finally concluded that it must be 90.25 because 25 was one quarter of a hundred. The length of this dialogue and the number of twists and turns that it took provided evidence that this was not a minor conflict for them, yet they did feel that it was within their competence to solve.

Conflicts that involved change to both the incoming fact and the existing scheme: These reactions to conflicts involved changes that allowed the new fact to be understood in terms of the students' existing schemes because the existing scheme was changed or expanded in some manner and the new fact was understood in a different way. Students with this reaction appeared to be adding a missing fragment or piece of a puzzle that made their scheme make sense. A marked example of this was H's realisation that the reason why $.9 + .1 = 1.0$ was that .9 meant $\frac{9}{10}$ and .1 meant $\frac{1}{10}$ and $\frac{9}{10}$ and $\frac{1}{10}$ made one whole. In adding this missing fragment she both adapted her understanding of the presented facts and her wider scheme for understanding numerical relationships, so that both common fractions and decimal fractions were included in her understanding. The resolution of this conflict starts with an excited exclamation:

> H ... OH!! I know why now! Ten tenths is one whole. (pause)
> A Yeah!
> H Ten tenths is one whole.
> A Yeah.
> H So obviously point nine and point one is ten tenths, that's one whole, where as, give me a piece of paper?
> K You can write on that.
> H [writes and says] Point 9 equals nine tenths, and point 1 equals 1 tenth which equals to ten tenths, 1 whole.

$$.9 = {}^{9}\!/_{10} \quad .1 = {}^{1}\!/_{10} \quad = {}^{10}\!/_{10} \quad 1$$

Responses to Conflict That Involve a Flexible System (Gamma Reactions)

This category is one in which there is no perturbation because the learner's overall scheme is already flexible enough to accommodate the new fact. The difference between this category and alpha (i) reactions appears to be in the flexibility of the learner's overall scheme. An example of a larger more flexible scheme that appeared to be developed within the time span of this study was provided by H, who talked with her partner a great deal. She had worked out that 2.30 x 10 would be 23 by adding 2.30 ten times, after which she noted the calculator answer of 23 and said:

> "You take out that zero, you get 2.3, and if you timesed it by 10 you just put... 2.3 and then times it by 10, you put the dot one place up, one place over the three, instead of behind it, over the three."

Later she guessed that $1\frac{1}{2}$ x 10 would be 10.5 and when the calculator gave 15 as the answer she said, "I get it now....You just take the decimal point from the 1 point five and just put it in front of the 5."

By the posttest she had solidified her knowledge into a rule, perhaps as a result of talking about it with an adult. If she did ask an adult about it, then this would be an example of a conflict providing the motivation for seeking out further knowledge. The question on the posttest and her response were:

> (Q) Give a rule for multiplying or dividing decimal fractions by 10. Give an example.
>
> (H) (writes) To multiply a decimal number(s) by 10 you bring the decimal point one place further to get the answer. i.e. 1.5 x 10 = 15. To divide a decimal number(s) by 10 you take the decimal point back one place in your answer. i.e. 15. ÷ 10 = 1.5

None of the other students was able to produce a rule like this.

Summary

In this study some students showed a higher proportion of beta and gamma reactions, or reactions that led to greater understanding. They were usually the students who had more knowledge initially. In Glaser's terms (1984) the more knowledgeable reasoned more knowledgeably. This was not only the higher-ranked students in the study. Lower-ranked students who dealt with contextualized problems had out-of-school knowledge to bring to the problem. This enabled them to also reason knowledgeably, in beta reactions, and thus expand their understanding. Nearly all the conflicts presented to pairs who worked on contextualized problems were resolved with either beta or gamma reactions, adding a fragment, compartmentalising, integrating, or seeing how new facts fitted into a flexible system.

This study suggest that teachers can both look for and engineer conflicts to help students construct their understanding. Having students work in pairs increases the likelihood of conflicts arising that they will be able to resolve. These conflicts are most productive if based on at least one aspect of knowledge that students know securely. They can then use this knowledge for solving the problem or resolving the conflict. At the same time a teacher needs to make certain that conflicts are within students' ability to resolve, so that they do not build up negative emotions about mathematics.

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